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${}^1\boldsymbol{P}_1$ charmonium state decay into $\boldsymbol{p\,\bar{p}}$ in QCD models including constituent quark mass corrections

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Abstract

Stimulated by the experimental observation, made by the E760 Collaboration at Fermilab, of the ${}^1P_1$ state of charmonium resonantly formed in $p\,\bar{p}$ annihilation, we perform a calculation of the decay width for the ${}^1P_1 \to p\,\bar{p}$ process. To this end, we employ a phenomenological model which adds constituent quark mass corrections to the usual massless QCD models for exclusive processes. For massless models, in fact, the process under consideration is forbidden by the so-called helicity selection rules, while it is allowed in our extended model. We find $\Gamma({}^1P_1 \to p\,\bar{p})$ to be in the range $1 - 10$ eV. We also compare our results with previous, indirect estimates, based on QCD multipole expansion models.

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I. INTRODUCTION

Recently the E760 Collaboration at Fermilab [1] have reported the first possible observation of the $^1P_1$ bound state of charmonium (this resonance, which possibly needs further confirmation, has been indicated as $h_c(1P)$ in the last Review of Particle Properties [2]). The experimental study of the $^1P_1$ charmonium state is very interesting for testing QCD theoretical models. The position of its mass with respect to the center of gravity of the $\chi_c(^3P_J)$ states provides a way to test different potential models for the $c\,\bar{c}$ bound state. Furthermore, the branching ratios for the $^1P_1$ hadronic exclusive decays relate to the validity of the so-called QCD helicity selection rules (HSR) [3] and of the QCD multipole expansion models (see, e.g., Ref. [4] and references therein). In particular, the $^1P_1 \to p\,\bar{p}$ decay is forbidden by the HSR's in massless QCD models for exclusive hadronic processes at high transfer momentum [3] and, due to time reversal invariance, the same should hold for the exclusive formation process $p\,\bar{p} \to {}^1P_1$, in contradiction with the results of Ref. [1], if the identification of the resonance observed at Fermilab with the $^1P_1$ state is correct.

As a matter of fact, while massless QCD models compare reasonably well with experimental measurements in several cases, they are in disagreement with the data for a number of processes, particularly with those violating the HSR's. We must bear in mind, however, that these models and their main qualitative results, like the QCD Counting Rules [3] or the HSR's, are in principle reliable at very high transfer momentum, while the available experimental results are all in a range of energies where their full validity can be at least questionable.

Several higher twist effects can in principle be responsible for the violation of the HSR's. If implemented in the models, they could give, at least in some cases, a way to reconcile theoretical predictions and experimental measurements. Unfortunately, a consistent inclusion of all higher twist effects is far from being a trivial work. In the last years, however, a few extensions of massless QCD models have been proposed, in order to study the possible role played by some of these higher twist effects. As an example, the diquark model for baryons is one of these (see, e.g., Ref. [5] for a general overview on this topic). In previous work [6–8] we have instead proposed a different, phenomenological model that extends the original massless QCD models by including constituent quark mass effects for the light (as compared to the transfer momentum scale in the process) hadrons produced in the final state. In this model the valence quarks of the final hadrons are given a (constituent) mass, $m_q = x_q m_H$, where x_q is the light-cone fraction of the hadron momentum carried by the quark. In the small-intermediate Q^2 region, in fact, one may think that the valence, constituent quarks (the current quarks surrounded by their cloud of $q\,\bar{q}$ pairs and gluons) still act as a single, effective particle. As shown by Weinberg [9], these constituent quarks can indeed be treated as bare Dirac particles, with the same coupling as for current quarks in the standard model QCD Lagrangian.

This model introduces spin-flip contributions in the elementary amplitudes for the hard scattering process, and as a consequence violations of the HSR's are in principle allowed. This is possibly the simplest extension of the ordinary massless models, and can be easily compared with them. It is assumed to be useful in the intermediate range of momentum transfer Q^2 where, like in charmonium decays, the use of QCD models is reasonable but at the same time higher twist effects, of the order m_H^2/Q^2 (m_H being a hadronic mass scale),

may still play an important role. As the squared momentum transfer Q^2 increases, the predictions of the model smoothly tend to those of the usual massless models.

This model has been previously applied to several interesting processes [6–8], in order to test the role of higher twist effects at the scale of momentum transfer presently accessible by experiments. For example, it predicts a decay width for the HSR-violating process $\chi_{c0} \to p\bar{p}$ comparable with those relative to the HSR-allowed decays $\chi_{c1,c2} \to p\bar{p}$ [6] (however, only a huge experimental upper limit is available for $\Gamma(\chi_{c0} \to p\bar{p})$, so that this prediction cannot be severely tested at present); it also reproduces satisfactorily the experimental angular distribution of the $p\bar{p}$ pair in the decay $J/\psi \to p\bar{p}$, where the role of HSR-violating contributions is essential (see Ref. [8] and references therein).

However, we must stress that other nonperturbative effects than the higher twist effects implemented in the model proposed here can also be relevant for charmonium decays. In Ref. [6], e.g., it has been shown that constituent mass corrections to the usual perturbative QCD models are not sufficient to explain the large experimental width for the HSR-violating decay $\eta_c \to p\bar{p}$. In fact, the η_c state seems to be very peculiar, and in order to explain its, somehow surprisingly, large decay widths into $p\bar{p}$ or vector meson pairs, nonperturbative explanations, like strong gluonic components [10] or instanton-induced contributions [11], have been proposed. The so-called $\rho\pi$ puzzle [3] and the radiative decays of the J/ψ [12] offer other examples of processes where nonperturbative effects can play a substantial role.

It is well possible that different higher twist or nonperturbative effects, like those quoted above, affect charmonium decays, each of them being perhaps more relevant for some specific process and less for others. A detailed comparison of different, theoretical models with all the available experimental results is then required to improve our present understanding of charmonium decays.

Following this line of research, in this paper we apply the model of Ref. [6–8] to the calculation of the decay width for the charmonium decay process ${}^1P_1 \to p\,\bar{p}$, whose interest has been briefly recalled before.

The plan of the paper is the following: in Sect. II we present the derivation of the decay width for the process ${}^1P_1 \to p\,\bar{p}$ in the framework of QCD models including constituent quark mass effects. Since this can also be useful as a general example of how these kind of calculations are performed, we will give a detailed presentation of all the crucial steps of the calculation. In Sect. III we present the results in a form which possibly exploit the model at its best, minimizing whenever possible model ambiguities. Finally, in Sect. IV we give some final comments and remarks.

II. EVALUATION OF THE DECAY WIDTH FOR THE PROCESS ${}^1P_1 \to p\,\bar{p}$

The general expression of the differential decay width for the process ${}^1P_1 \to p\,\bar{p}$ is the following:

$$\frac{d\Gamma({}^1P_1 \to p\bar{p})}{d\phi d(\cos\theta)} = \frac{1}{8(2\pi)^5}(1-4\epsilon_1^2)^{1/2} \sum_{M,M',\lambda,\lambda'} \rho_{MM'}({}^1P_1) \times A_{\lambda,\lambda';M}(\theta,\phi) A^*_{\lambda,\lambda';M'}(\theta,\phi) \;, \tag{1}$$

where θ and ϕ are respectively the polar and azimuthal angles defining the outgoing direction of the final proton, in the ${}^1P_1$ rest frame; ϵ_1 is the ratio between the proton mass, m_p, and the ${}^1P_1$ mass, M_1; $\rho({}^1P_1)$ is the spin density matrix of the ${}^1P_1$; finally, $A_{\lambda,\lambda';M}(\theta,\phi)$ is the rest frame helicity amplitude for the decay of the ${}^1P_1$ bound state, with z component M of its total angular momentum, $J=1$, into a proton and an antiproton whose helicities are respectively λ and λ'.

It is known from first principles [13] that the amplitudes $A_{\lambda\lambda';M}(\theta,\phi)$ have the following general structure:

$$A_{\lambda\lambda';M}(\theta,\phi) = \tilde{A}_{\lambda\lambda'}\, d^1_{M,\lambda-\lambda'}(\theta)\, \exp(iM\phi)\ , \tag{2}$$

where the "reduced" amplitude $\tilde{A}_{\lambda\lambda'}$ is independent of M and the angular variables and the $d^{\,J}(\theta)$ are the usual spin rotation matrices.

Inserting Eq. (2) into Eq. (1) and integrating over $d\phi$ we get, since $\int_0^{2\pi} d\phi \exp[i(M-M')\phi] = 2\pi\delta(M-M')$,

$$\begin{aligned}\frac{d\Gamma({}^1P_1\to p\bar{p})}{d(\cos\theta)} &= \frac{1}{8(2\pi)^4}(1-4\epsilon_1^2)^{1/2}\sum_{\lambda,\lambda'}|\tilde{A}_{\lambda\lambda'}|^2 \\ &\times \sum_M \rho_{MM}({}^1P_1)\left[d^1_{M,\lambda-\lambda'}(\theta)\right]^2\ .\end{aligned} \tag{3}$$

Integrating now over $d(\cos\theta)$ and using the facts that $\int_0^{\pi} d(\cos\theta)\left[d^J_{\mu\mu'}(\theta)\right]^2 = 2/(2J+1)$, whatever μ and μ' are and that, by parity invariance, $|\tilde{A}_{--}|^2 = |\tilde{A}_{++}|^2$ and $|\tilde{A}_{-+}|^2 = |\tilde{A}_{+-}|^2$, we finally find:

$$\Gamma({}^1P_1\to p\bar{p}) = \frac{1}{96\pi^4}(1-4\epsilon_1^2)^{1/2}\left[|\tilde{A}_{++}|^2 + |\tilde{A}_{+-}|^2\right]\ . \tag{4}$$

Before proceeding with calculations, let us remind why massless QCD models predict a vanishing decay width for the process under consideration. First of all, amplitudes with the proton and antiproton having opposite helicities, $\tilde{A}_{\lambda,-\lambda}$, must vanish: in fact, the initial state of the process, the ${}^1P_1$, is a $J^{PC}=1^{+-}$ bound state; on the other hand, the outgoing $p\,\bar{p}$ pair is, for $\tilde{A}_{\lambda,-\lambda}$, in a $S=1$ state. Then, in order for the pair to have charge conjugation $C=(-1)^{L+S}=-1$, its relative angular momentum L should be even, contradicting the requirement imposed by parity conservation, $P=(-1)^{L+1}$, that L should be odd [14]. The crucial point is now that in massless QCD models helicity amplitudes with the proton and the antiproton having the same helicity, $\tilde{A}_{\lambda,\lambda}$, are also vanishing, due to the HSR's, which forbid spin-flips in the elementary quark-gluon vertices. Then there is not any term contributing to the total decay width, Eq. (4). Notice that HSR's are valid to all orders in the perturbative expansion in the strong coupling constant, and as such overcoming their restrictions is not a matter of going to higher order in this expansion. On the contrary, every higher twist contribution violating the HSR's would in principle give a non vanishing contribution to $\tilde{A}_{\lambda,\lambda}$, of the order m_H/M_1, where m_H is a typical hadronic scale.

We now proceed considering in more details the helicity amplitudes $A_{\lambda\lambda';M}(\theta,\phi)$ which enter the calculation of the $\Gamma({}^1P_1\to p\,\bar{p})$ decay width. Let us define: *i)* $A^{L\,S\,J}_{\lambda,\lambda';M}(\theta,\phi)$ as the helicity amplitude for the decay of a $c\,\bar{c}$ bound state with quantum numbers L,S,J,M, into

a $p\,\bar{p}$ pair with helicities λ, λ' respectively; *ii)* $M_{\lambda\lambda';\lambda_c\lambda_{\bar{c}}}(\boldsymbol{k};\theta,\phi)$ as the helicity amplitude for the decay of two free c, $\bar{c}$ quarks with relative momentum $\boldsymbol{k}$ and helicities λ_c, $\lambda_{\bar{c}}$ into the same final state as before. Then, in the well-known nonrelativistic approximation, $A^{L\,S\,J}_{\lambda,\lambda';M}$ can be obtained by integrating the amplitude $M_{\lambda\lambda';\lambda_c\lambda_{\bar{c}}}$ over the proper $c\,\bar{c}$ bound state, nonrelativistic wave function:

$$
\begin{aligned}
A^{L\,S\,J}_{\lambda,\lambda';M}(\theta,\phi) = &\sum_{\lambda_c\lambda_{\bar{c}}}\left(\frac{2L+1}{4\pi}\right)^{1/2} C^{\frac{1}{2}\ \frac{1}{2}\ S}_{\lambda_c,\,-\lambda_{\bar{c}}\,\lambda}\, C^{L\,S\,J}_{0\,\lambda\,\lambda} \\
&\times\int d^3\boldsymbol{k}\, M_{\lambda\lambda';\lambda_c\lambda_{\bar{c}}}(\boldsymbol{k};\theta,\phi) D^{J\,*}_{M\lambda}(\beta,\alpha,0)\,\psi_C(k)\ ,
\end{aligned}
\tag{5}
$$

where the C's are the Clebsh-Gordan coefficients required to give the right combination of quantum numbers; $\lambda=\lambda_c-\lambda_{\bar{c}}$; $\boldsymbol{k}=(k,\alpha,\beta)$ is the relative momentum between the c and $\bar{c}$ quarks and finally $\psi_c(k)$ is the (momentum-space) charmonium wave function. In particular, for $L=1$ bound states, and taking a full nonrelativistic approximation (that is, $\boldsymbol{k}\to 0$),

$$
\psi_{L=1}(k) = i3\sqrt{2\pi}|R'_1(0)|\frac{1}{k^3}\delta(k)\ ,
\tag{6}
$$

where $|R'_1(0)|$ is the absolute value of the first derivative of the bound state radial wave function at the origin.

Using Eqs. (2,5,6), and defining $u=\cos\alpha$ we get:

$$
\begin{aligned}
&A_{++;0}(\theta=\phi=0) = \tilde{A}_{++} = \\
&i\frac{3^{3/2}}{2}|R'_1(0)|\lim_{\boldsymbol{k}\to 0}\frac{1}{k}\int_0^{2\pi}\!d\beta\int_{-1}^{+1}\!du\ u \\
\times&\left[M_{++;++}(\boldsymbol{k};\theta\!=\!\phi\!=\!0) - M_{++;--}(\boldsymbol{k};\theta\!=\!\phi\!=\!0)\right]\ ,
\end{aligned}
\tag{7}
$$

and

$$
\begin{aligned}
&A_{+-;1}(\theta=\phi=0) = \tilde{A}_{+-} = \\
&-i\left(\frac{3}{2}\right)^{3/2}|R'_1(0)|\lim_{\boldsymbol{k}\to 0}\frac{1}{k}\int_0^{2\pi}\!d\beta\ e^{i\beta}\int_{-1}^{+1}\!du\left(1-u^2\right)^{1/2} \\
\times&\left[M_{++;++}(\boldsymbol{k};\theta\!=\!\phi\!=\!0) - M_{++;--}(\boldsymbol{k};\theta\!=\!\phi\!=\!0)\right]\ .
\end{aligned}
\tag{8}
$$

Notice that from now on we restrict our calculations to the simple kinematical configuration $\theta=\phi=0$, since the angular dependence of the helicity amplitudes $A_{\lambda,\lambda';M}$ can be deduced from Eq. (2).

In order to proceed further and evaluate the helicity amplitudes $M_{\lambda\lambda';\lambda_c\lambda_{\bar{c}}}(\boldsymbol{k};\theta,\phi)$ we resort to the previously mentioned QCD models [3]. According to them and to factorization theorems, the amplitudes M are given, under particular conditions that are reasonably applicable to charmonium decays, by a convolution between a hard, perturbative contribution, describing the interaction between the $c\,\bar{c}$ pair and the valence quarks of the final baryons, and a soft, non-perturbative contribution, which accounts for the hadronization of the valence quarks into the final hadrons. The latter contribution is described, for each final hadron, by the respective distribution amplitude (DA), which is in principle independent of

the particular process under consideration. As is well known, the exact form of the DA's is essential for the quantitative results of the model. Then we have:

$$
\begin{aligned}
M_{\lambda\lambda';\lambda_c\lambda_{\bar c}}(\boldsymbol{k},\theta,\phi) &= \int [d\tilde{x}][d\tilde{y}]\psi_{p,\lambda}(\tilde{x})\psi_{\bar{p},\lambda'}(\tilde{y}) \\
&\times T_{\lambda_{q_1}\lambda_{q_2}\lambda_{q_3},\lambda_{\bar q_1}\lambda_{\bar q_2}\lambda_{\bar q_3};\lambda_c\lambda_{\bar c}}(\tilde{x},\tilde{y};\boldsymbol{k},\theta,\phi)\ ,
\end{aligned}
\tag{9}
$$

where $\int[d\tilde{z}]$ stays for $\int_0^1 dz_1dz_2dz_3\delta(1-z_1-z_2-z_3)$ and we used $\tilde{z}$ as a shorthand for (z_1,z_2,z_3); $x_i(y_i)$ represents the (light-cone) fraction of the proton (antiproton) four-momentum carried by the i-th quark (antiquark); the $\psi(\tilde{z})$ are the baryon wave functions, containing a color, spin-flavor and dynamical part, the above mentioned DA. We will discuss in more details these wave functions in the following (see Eq. (22) and following comments). The helicity amplitude T describes the hard elementary scattering. To lowest order in the strong coupling constant there is only one Feynman graph contributing (apart from a set of similar terms obtained by permutations of the final fermionic lines, which can be accounted for by a proper choice of the hadron wave function). It is shown in Fig. 1, where the kinematical notation is also defined.

The hard scattering amplitude T has the following expression (we use $\{\lambda\}$ as a shorthand notation for the set of quark and antiquark helicities, $\lambda_{q_1}\lambda_{q_2}\lambda_{q_3},\lambda_{\bar q_1}\lambda_{\bar q_2}\lambda_{\bar q_3}$):

$$
\begin{aligned}
T_{\{\lambda\};\lambda_c\lambda_{\bar c}} &= -c_F(4\pi\alpha_s)^3\frac{1}{g_1^2g_2^2g_3^2}\frac{1}{(k_1^2-m_c^2)(k_2^2-m_c^2)} \\
&\times R_{(q)\mu\nu\rho}(\{\lambda\})R_{(c)}^{\mu\nu\rho}(\lambda_c,\lambda_{\bar c})\ ,
\end{aligned}
\tag{10}
$$

where c_F is the color factor which, once the convolution with the final hadron wave functions is made, takes the value $c_F = 5/(18\sqrt{3})$;

$$
g_i^2 = M_1^2[x_iy_i + (x_i-y_i)^2\epsilon_1^2] \tag{11}
$$

is the squared four-momentum of the i-th virtual gluon, $i=1,2,3$; $k_1 = c-g_1$ and $k_2 = g_3-\bar{c}$ are the four-momenta of the two virtual charm quarks (see Fig. 1); m_c is the charm quark mass (we assume, consistently with the adopted nonrelativistic approximation for the bound state, $m_c \simeq M_1/2$); moreover,

$$
(k_1^2-m_c^2)(k_2^2-m_c^2) = \frac{M_1^4}{4}(A+B\hat{k}u+C\hat{k}^2u^2)\ , \tag{12}
$$

where $\hat{k} = k/M_1$, and we have defined

$$
\begin{aligned}
A &= \prod_{i=1,3}\Big[2x_iy_i - x_i - y_i + 2(x_i-y_i)^2\epsilon_1^2\Big]\ ; \\
B &= \Big(1-4\epsilon_1^2\Big)^{1/2}\Big\{(x_1-y_1)\Big[2x_3y_3 - x_3 - y_3 \\
&\quad + 2(x_3-y_3)^2\epsilon_1^2\Big] - (1\longleftrightarrow 3)\Big\}\ ; \\
C &= -\Big(1-4\epsilon_1^2\Big)(x_1-y_1)(x_3-y_3)\ ;
\end{aligned}
\tag{13}
$$

finally

$$R_{(q)}^{\mu_1\mu_2\mu_3}(\{\lambda\}) = \prod_{i=1}^{3} \bar{u}(q_i,\lambda_{q_i})\gamma^{\mu_i} v(\bar{q}_i,\lambda_{\bar{q}_i}) \ , \tag{14}$$

and

$$\begin{aligned} R_{(c)}^{\mu\nu\rho}(\lambda_c,\lambda_{\bar{c}}) \ &= \ \bar{v}(\bar{c},\lambda_{\bar{c}})\gamma^{\mu}(\not{k}_2 + m_c) \\ &\quad \times \gamma^{\nu}(\not{k}_1 + m_c)\gamma^{\rho} u(c,\lambda_c) \ . \end{aligned} \tag{15}$$

Notice that Eq. (4) implies that the helicity spinors for the c, $\bar{c}$ quarks are normalized to one, while those for the final quarks are normalized to $2m$.

Inserting Eqs. (9),(10) into Eqs. (7),(8) we find:

$$\begin{aligned} \tilde{A}_{++} \ &= \ -i\frac{3^{3/2}}{2}\,|R_1'(0)|\,c_F\,(4\pi)^3\int_0^1 [d\tilde{x}]\,[d\tilde{y}]\,\psi_{p,+}(\tilde{x}) \\ &\quad \times \psi_{\bar{p},+}(\tilde{y})\left[\alpha_s^3\frac{1}{g_1^2g_2^2g_3^2}R_{(q)\mu\nu\rho}\right]_{k=0}\hat{F}_{(0)}^{\mu\nu\rho} \ ; \end{aligned} \tag{16}$$

$$\begin{aligned} \tilde{A}_{+-} \ &= \ i\left(\frac{3}{2}\right)^{3/2}|R_1'(0)|\,c_F\,(4\pi)^3\int_0^1 [d\tilde{x}]\,[d\tilde{y}]\,\psi_{p,+}(\tilde{x}) \\ &\quad \times \psi_{\bar{p},-}(\tilde{y})\left[\alpha_s^3\frac{1}{g_1^2g_2^2g_3^2}R_{(q)\mu\nu\rho}\right]_{k=0}\hat{F}_{(1)}^{\mu\nu\rho} \ , \end{aligned} \tag{17}$$

where we have defined ($n = 0, 1$)

$$\begin{aligned} \hat{F}_{(n)}^{\mu\nu\rho} \ &= \ \frac{4}{M_1^5}\lim_{\hat{k}\to 0}\frac{1}{\hat{k}}\int_{-1}^{+1}du\,\frac{u^{1-n}\,(1-u^2)^{n/2}}{A + B\hat{k}u + C\hat{k}^2u^2} \\ &\quad \times \int_0^{2\pi}d\beta\,\exp(in\beta)\,F^{\mu\nu\rho} \ \ , \end{aligned} \tag{18}$$

and

$$F^{\mu\nu\rho} = R_{(c)}^{\mu\nu\rho}(+,+) - R_{(c)}^{\mu\nu\rho}(-,-) \ . \tag{19}$$

Notice that in Eqs. (16),(17) the strong coupling constant α_s has been left inside the convolution integral. In fact, strictly speaking, its (running) value depends on the momentum fractions x_i, y_i carried by the quarks participating to the hard scattering, as will be discussed in the next section. Furthermore, the color part of the baryon wave functions $\psi_{p,\bar{p}}$ is now included into the color factor c_F.

After some length but straightforward algebra we get

$$\begin{aligned} \tilde{A}_{++} \ &= \ i2^9\sqrt{3}|R_1'(0)|c_F\pi^4\frac{1}{M_1^9}\int_0^1 [d\tilde{x}]\,[d\tilde{y}]\,\psi_{p,+}(\tilde{x})\psi_{\bar{p},+}(\tilde{y})\prod_{i=1}^{3}\frac{\alpha_s(g_i^2)}{x_iy_i + (x_i-y_i)^2\epsilon_1^2}\frac{1}{A} \\ &\times \left\{\frac{B}{A}\left[\left[x_1y_3 + x_3y_1 + 2(x_1-y_1)(x_3-y_3)\epsilon_1^2\right]\left[R_{(q)123} - R_{(q)213} + R_{(q)312} - R_{(q)321}\right]\right.\right. \\ &- \left.\left[x_1x_3 + y_1y_3 - 2(x_1-y_1)(x_3-y_3)\epsilon_1^2\right]\left[R_{(q)132} - R_{(q)231}\right]\right] \\ &+ \left.\left(1-4\epsilon_1^2\right)^{1/2}\left[(x_1-y_1)(R_{(q)312} - R_{(q)321}) - (x_3-y_3)(R_{(q)123} - R_{(q)213})\right]\right\} \ ; \end{aligned} \tag{20}$$

$$
\begin{aligned}
\tilde{A}_{+-} &= i2^8\sqrt{6}|R_1'(0)|c_F\pi^4(1-4\epsilon_1^2)^{1/2}\frac{1}{M_1^9}\int_0^1[d\tilde{x}]\,[d\tilde{y}]\,\psi_{p,+}(\tilde{x})\psi_{\bar{p},-}(\tilde{y}) \\
&\times\prod_{i=1}^{3}\frac{\alpha_s(g_i^2)}{x_iy_i+(x_i-y_i)^2\epsilon_1^2}\frac{1}{A}\Big\{(x_1-y_1)\left[R_{(q)112}-iR_{(q)221}\right] \\
&+(x_2-y_2)\left[R_{(q)121}-iR_{(q)212}\right]+(x_3-y_3)\left[R_{(q)211}-iR_{(q)122}\right]\Big\}\ . \qquad (21)
\end{aligned}
$$

Considering now the explicit expression of the most general spin-flavor component of the proton wave function (see, e.g., Ref. [15] and references therein),

$$
\begin{aligned}
\psi_{p,\lambda}(\tilde{x}) = 2\lambda\frac{F_N}{4\sqrt{6}}\{&\varphi(123)u_{1,\lambda}(1)u_{2,-\lambda}(2)d_{3,\lambda}(3)+\varphi(213)u_{1,-\lambda}(1)u_{2,\lambda}(2)d_{3,\lambda}(3) \\
&-2T(123)u_{1,\lambda}(1)u_{2,\lambda}(2)d_{3,-\lambda}(3)+(1\longleftrightarrow 3)+(2\longleftrightarrow 3)\ \}\ , \qquad (22)
\end{aligned}
$$

where F_N is the so-called nucleon decay constant, $\varphi(\tilde{x})$ is the distribution amplitude, $2T(123)=\varphi(132)+\varphi(231)$ (in obvious notation, $\varphi(ijk)=\varphi(z_i,z_j,z_k)$), we find that, being $\Phi(\tilde{x},\tilde{y})$ a generic function of $\tilde{x}$, $\tilde{y}$,

$$
\begin{aligned}
\int[d\tilde{x}]\,[d\tilde{y}]\ \ &\psi_{p,+}(\tilde{x})\psi_{\bar{p},+}(\tilde{y})\Phi(\tilde{x},\tilde{y})(R_{(q)123}-R_{(q)213}) = \frac{F_N^2}{24}M_1^3\epsilon_1\int[d\tilde{x}]\,[d\tilde{y}]\,\Phi(\tilde{x},\tilde{y}) \\
&\times\ \Big\{\varphi(123)\varphi(213)-\varphi(213)\varphi(123)+2\varphi(312)T(132) \\
&-2T(132)\varphi(312)-2\varphi(321)T(321)+2T(321)\varphi(321)\Big\}\ ; \qquad (23)
\end{aligned}
$$

$$
\begin{aligned}
\int[d\tilde{x}]\,[d\tilde{y}]\ \ &\psi_{p,+}(\tilde{x})\psi_{\bar{p},+}(\tilde{y})\Phi(\tilde{x},\tilde{y})(R_{(q)312}-R_{(q)321}) = \frac{F_N^2}{24}M_1^3\epsilon_1\int[d\tilde{x}]\,[d\tilde{y}]\,\Phi(\tilde{x},\tilde{y}) \\
&\times\ \Big\{2\varphi(123)T(123)-2T(123)\varphi(123)-2\varphi(132)T(132) \\
&+2T(132)\varphi(132)-\varphi(321)\varphi(231)+\varphi(231)\varphi(321)\Big\}\ ; \qquad (24)
\end{aligned}
$$

$$
\begin{aligned}
\int[d\tilde{x}]\,[d\tilde{y}]\ \ &\psi_{p,+}(\tilde{x})\psi_{\bar{p},+}(\tilde{y})\Phi(\tilde{x},\tilde{y})(R_{(q)132}-R_{(q)231}) = \frac{F_N^2}{24}M_1^3\epsilon_1\int[d\tilde{x}]\,[d\tilde{y}]\,\Phi(\tilde{x},\tilde{y}) \\
&\times\ \Big\{2\varphi(213)T(123)-2T(123)\varphi(213)+\varphi(132)\varphi(312) \\
&-\varphi(312)\varphi(132)-2\varphi(231)T(321)+2T(321)\varphi(231)\Big\}\ ; \qquad (25)
\end{aligned}
$$

$$
\begin{aligned}
\int[d\tilde{x}]\,[d\tilde{y}]\,&\psi_{p,+}(\tilde{x})\psi_{\bar{p},-}(\tilde{y})\Phi(\tilde{x},\tilde{y})\left(R_{(q)112}-iR_{(q)221}\right) = -\frac{F_N^2}{48}M_1^3\int[d\tilde{x}]\,[d\tilde{y}]\,\Phi(\tilde{x},\tilde{y}) \\
&\times\Big\{\varphi^2(123)+\varphi^2(213)-4T^2(123)-\varphi^2(132)+\varphi^2(312)+4T^2(132) \\
&+\varphi^2(321)-\varphi^2(231)+4T^2(321)\Big\}\ ; \qquad (26)
\end{aligned}
$$

$$
\begin{aligned}
\int[d\tilde{x}]\,[d\tilde{y}]\,&\psi_{p,+}(\tilde{x})\psi_{\bar{p},-}(\tilde{y})\Phi(\tilde{x},\tilde{y})\left(R_{(q)121}-iR_{(q)212}\right) = -\frac{F_N^2}{48}M_1^3\int[d\tilde{x}]\,[d\tilde{y}]\,\Phi(\tilde{x},\tilde{y}) \\
&\times\Big\{-\varphi^2(123)+\varphi^2(213)+4T^2(123)+\varphi^2(132)+\varphi^2(312)-4T^2(132) \\
&-\varphi^2(321)+\varphi^2(231)+4T^2(321)\Big\}\ ; \qquad (27)
\end{aligned}
$$

$$\int [d\tilde{x}]\,[d\tilde{y}]\;\psi_{p,+}(\tilde{x})\psi_{\bar{p},-}(\tilde{y})\Phi(\tilde{x},\tilde{y})\left(R_{(q)211} - iR_{(q)122}\right) = -\frac{F_N^2}{48}M_1^3 \int [d\tilde{x}]\,[d\tilde{y}]\;\Phi(\tilde{x},\tilde{y})$$
$$\times \Big\{\varphi^2(123) - \varphi^2(213) + 4T^2(123) + \varphi^2(132) - \varphi^2(312) + 4T^2(132)$$
$$+ \varphi^2(321) + \varphi^2(231) - 4T^2(321)\Big\}\,. \qquad (28)$$

In the products of the φ, T proton distribution amplitudes appearing in the previous equations, the first term is always intended to be a function of $\tilde{x}$, the second of $\tilde{y}$, while the number in brackets give the pertinent permutation of the z_1, z_2, z_3 arguments. It is important to notice, as we will see in a moment, that in all the terms concerning the $\tilde{A}_{++}$ (respectively $\tilde{A}_{+-}$) amplitude the contribution to the convolution integral coming from the proton and antiproton distribution amplitudes is totally antisymmetric (symmetric) under the exchange of $\tilde{x}$ and $\tilde{y}$.

Using Eqs. (20),(23)-(25), we finally find, after some simple algebra:

$$\begin{aligned}
\tilde{A}_{++} \;=\;& i\frac{5\cdot 2^6}{3^3}\pi^4\epsilon_1(1-4\epsilon_1^2)^{1/2}F_N^2|R_1'(0)|\frac{1}{M_1^6}\int [d\tilde{x}]\,[d\tilde{y}]\prod_{i=1}^{3}\frac{\alpha_s(g_i^2)}{x_iy_i + (x_i-y_i)^2\epsilon_1^2} \\
&\times \frac{1}{2x_1y_1 - x_1 - y_1 + 2(x_1-y_1)^2\epsilon_1^2}\,\frac{1}{2x_3y_3 - x_3 - y_3 + 2(x_3-y_3)^2\epsilon_1^2} \\
&\times \Bigg\{\Bigg[\frac{x_1-y_1}{2x_1y_1 - x_1 - y_1 + 2(x_1-y_1)^2\epsilon_1^2} - \frac{x_3-y_3}{2x_3y_3 - x_3 - y_3 + 2(x_3-y_3)^2\epsilon_1^2}\Bigg] \\
&\times \Bigg[\Big[x_1y_3 + x_3y_1 + 2(x_1-y_1)(x_3-y_3)\epsilon_1^2\Big]\Big[\varphi(123)\varphi(213) + 2\varphi(312)T(132) \\
&- 2\varphi(321)T(321) + 2\varphi(123)T(123) - 2\varphi(132)T(132) + \varphi(231)\varphi(321)\Big] \\
&- \Big[x_1x_3 + y_1y_3 - 2(x_1-y_1)(x_3-y_3)\epsilon_1^2\Big] \\
&\times \Big[2\varphi(213)T(123) + \varphi(132)\varphi(312) - 2\varphi(231)T(321)\Big]\Bigg] \\
&+ (x_1-y_1)\Big[2\varphi(123)T(123) - 2\varphi(132)T(132) + \varphi(231)\varphi(321)\Big] \\
&- (x_3-y_3)\Big[\varphi(123)\varphi(213) + 2\varphi(312)T(132) - 2\varphi(321)T(321)\Big]\Bigg\}\,. \qquad (29)
\end{aligned}$$

Notice that the $\tilde{A}_{++}$ amplitude is proportional to the ratio $\epsilon_1 = m_p/M_1$, so that, as it must be, it vanishes in the massless case, that is, in the $(m_q^2/Q^2) \sim (m_p^2/Q^2) \to 0$ limit (remember that in our model $m_q = x_q m_p$, x_q being the light-cone fraction of the hadron momentum carried by the quark).

It is also easy to verify explicitly that, as was discussed previously, the amplitude $\tilde{A}_{+-}$ is vanishing, as it must be from first principles. In fact, Eqs. (21),(26)–(28) show that the argument of the integral over $d\tilde{x}$, $d\tilde{y}$ is totally antisymmetric under the exchange of $\tilde{x}$ and $\tilde{y}$.

III. RESULTS

We are now equipped with all the ingredients required to perform estimates of the decay width for the process ${}^1P_1 \to p\,\bar{p}$.

To begin with, let us recall very briefly some aspects of QCD models (both in the massless and massive cases) which are essential in determining their quantitative predictions. As previously mentioned, a very important component of the calculations are the DA's. The so-called asymptotic DA's, as deducible from perturbative QCD at very large values of the transfer momentum, are not guaranteed to be reliable in the range of energies experimentally accessible today and presumably also in the near future, due to the their slow, logarithmic evolution with Q^2. Non-perturbative methods, like Lattice QCD or QCD sum rules, give information on the first moments of the DA's that lead to a number of models, widely used for practical calculations. We will consider several of these model DA's, in order to get indications on the spread of our results due to this ingredient, without giving prominence to one model over the others.

The value of the first derivative of the nonrelativistic, radial bound state wave function is another parameter to be fixed. One may use potential models to estimate this quantity or, alternatively, fix its value phenomenologically. This can be done by comparing the theoretical prediction with the experimental measurement for some other process which is better understood theoretically and more accessible experimentally.

Sudakov factors can also change sizably the quantitative predictions. The inclusion of these factors has been advocated in order to improve the consistency of the theoretical models for exclusive processes and answer some criticisms connected with the end-point contributions (see Ref. [16] and references therein).

Furthermore, we do not know whether and how the DA's are modified due to higher twist, mass correction effects.

In order to improve the significance of our results and, where possible, to minimize their dependence on all the abovementioned uncertainties, we choose to evaluate, rather than the absolute value of the $\Gamma({}^1P_1 \to p\,\bar{p})$ decay width, its ratio with the decay width for the analogous process, $\chi_{c2} \to p\,\bar{p}$. In other words, we write

$$\Gamma({}^1P_1 \to p\,\bar{p}) = \left[\frac{\Gamma({}^1P_1 \to p\,\bar{p})}{\Gamma(\chi_{c2} \to p\,\bar{p})}\right] \Gamma(\chi_{c2} \to p\,\bar{p}) \; , \tag{30}$$

where we use the theoretical prediction for $\Gamma(\chi_{c2} \to p\,\bar{p})$ in the denominator and the corresponding experimental measurement in the numerator of the r.h.s. of Eq. (30).

This way, we can neglect the dependence of the results on F_N and $|R_1'(0)|$. Notice that F_N and $|R_1'(0)|$ appear in the expression of $\Gamma({}^1P_1 \to p\,\bar{p})$ to the 4th and 2nd power respectively, so that even small variations in their values might give relatively sizable changes in our observable. Possible Sudakov factor effects might also be mitigated in the ratio. We are then confident that our numerical estimates for the $\Gamma({}^1P_1 \to p\,\bar{p})$ decay width, given in the form of Eq. (30), have the higher precision attainable with present models.

The decay width $\Gamma(\chi_{c2} \to p\,\bar{p})$ has been evaluated previously in massless QCD models (see Ref. [15] and references therein). We have done the calculation following the same procedure previously described for the $\Gamma({}^1P_1 \to p\,\bar{p})$ case. Our result is in agreement with

that of Ref. [15]. Notice that this calculation accounts only for the leading term in a (m_p/M) power expansion (in that case the $(m_p/M)^0$ term, since the process is not forbidden by HSR's). For consistency, we also keep only the leading, (m_p/M), term in the calculation of $\Gamma(^1P_1 \to p\,\bar{p})$. It is important to notice that, this way, we avoid also possible problems connected with modifications of the DA's when mass corrections are taken into account, since these modifications do not contribute at leading order.

Using Eq. (4), and Eq. (29) at leading order in ϵ_1 we have:

$$\begin{aligned}\Gamma(^1P_1 \to p\,\bar{p}) &= \left(\frac{2}{3}\right)^7 5^2\pi^4\epsilon_1^2\left(1-4\epsilon_1^2\right)^{1/2} \\ &\quad \times F_N^4|R_1'(0)|^2\frac{1}{M_1^{12}}I_1^2\ ,\end{aligned} \tag{31}$$

where I_1 is the integral appearing in Eq. (29), taken in the limit $\epsilon_1 \to 0$. Notice that we have retained the mass contribution in the kinematical factor coming from phase space integration, $(1-4\epsilon_1^2)^{1/2}$.

The corresponding expression for the $\chi_{c2} \to p\,\bar{p}$ decay is [15,17]

$$\begin{aligned}\Gamma(\chi_{c2} \to p\,\bar{p}) &= \frac{2^{11}}{3^6\cdot 5}\pi^4\left(1-4\epsilon_2^2\right)^{1/2} \\ &\quad \times F_N^4|R_1'(0)|^2\frac{1}{M_2^{12}}I_2^2\ ,\end{aligned} \tag{32}$$

where M_2 is the mass of the χ_{c2} and $\epsilon_2 = m_p/M_2$. For completeness, we report the expression of the integral I_2 (see also Ref. [15])

$$\begin{aligned}I_2 &= \int [d\tilde{x}]\,[d\tilde{y}]\,\alpha_s[x_1y_1M^2]\alpha_s[x_3y_3M^2]\alpha_s[(1-x_1)(1-y_1)M^2] \\ &\quad \times \frac{x_1+y_1}{x_1y_1x_3y_3(1-x_1)(1-y_1)\left[x_1(1-y_1)+y_1(1-x_1)\right]^2} \\ &\quad \times \left\{\varphi^2(231)+\varphi^2(132)+\varphi^2(123)+\varphi^2(321)+4T^2(123)+4T^2(132)\right\}\ .\end{aligned} \tag{33}$$

Inserting Eq.s (31),(32) into Eq. (30) we find finally:

$$\Gamma(^1P_1 \to p\,\bar{p}) = \frac{5^3}{2^4\cdot 3}\epsilon_1^2\left(\frac{1-4\epsilon_1^2}{1-4\epsilon_2^2}\right)^{1/2}\left(\frac{M_2}{M_1}\right)^{12}\left(\frac{I_1}{I_2}\right)^2\Gamma^{exp}(\chi_{c2} \to p\,\bar{p})\ . \tag{34}$$

In Table I we present the results obtained using Eq. (34) and the available models for the DA's. Besides the nonrelativistic (NR) and asymptotic (AS) DA's [3], we have used model DA's inspired by QCD sum rules methods: the Chernyak, Zhitnitsky (CZ) model and its improved (COZ) version, due to Chernyak, Ogloblin, and Zhitnitsky (see Ref. [15] and references therein); the King, Sachrajda (KS) model [18]; the Gari, Stefanis (GS) model [19]; and, finally, the Heterotic (HET) model proposed by Stefanis and Bergmann [20]. A collection of the explicit expressions of these DA's can be found in Ref. [8].

The expression used for the strong, running coupling constant inside the convolution integrals I_1 and I_2 is the following:

$$\alpha_s(Q^2) = \frac{12\pi}{(11n_c - 2n_f)\log\left[(Q^2+4m_g^2)/\Lambda^2\right]} \quad , \tag{35}$$

where $n_c = 3$ is the number of colors; n_f is the number of active flavors (in this context, $n_f = 4$); $m_g \sim 0.5$ GeV is an effective, dynamical gluon mass regularizing the α_s behavior at slow Q^2 [21]; we assume also $\Lambda = 0.2$ GeV/c.

The Q^2 evolution of the proton DA's, according to perturbative QCD, has been also taken into account.

Finally, we have used $\Gamma^{exp}(\chi_{c2} \to p\,\bar{p}) = (206 \pm 22)$ eV [2,22].

It is evident from Table I that, in spite of our efforts in order to limit the dependence of the numerical results on the particular shape of the model DA, our estimate for $\Gamma({}^1P_1 \to p\,\bar{p})$, even excluding the NR and AS distribution amplitudes, still cover a range from 1 to 10 eV. This spread of the results is in part due to the different behavior of the integrals I_1 and I_2 when changing the DA's. Relative changes are more sensitive for I_1 than for I_2. This can be easily understood from a qualitative point of view. In fact, the integral I_1 is exactly vanishing for DA's $\varphi(\tilde{z})$ that, like the nonrelativistic and the asymptotic ones, are completely symmetric under permutations of z_1, z_2, z_3. Moreover, due to the factors $(x_i - y_i)$ which are present in each contribution to I_1, I_1 itself is very depressed when $x_i \sim y_i$, as is the case for DA's that are strongly peaked at $x_i \sim y_i \sim 1/3$. Then, the magnitude of I_1 should give a measure of how much the corresponding DA is far from the simple, intuitive equipartition of the hadron momentum among its valence constituents. In principle this could give an interesting way to discriminate at least the gross features of the proton DA's. In practice, however, at this stage of the calculations, these effects are probably masked by other, less controlled, sources of uncertainty. From this point of view it should surely be of relevance to perform a full implementation of mass corrections. We have not calculated them for the $\chi_{c2} \to p\,\bar{p}$ process, while for the ${}^1P_1$ case, direct use of the full expression of $\tilde{A}_{++}$, Eq. (29), neglecting possible modifications in the shape of the DA, gives a sizable reduction in the total width. The worst case then would be if a full account of mass corrections in the χ_{c2} case should give an enhancement of the corresponding decay width, depressing our prediction for the relative $\Gamma({}^1P_1 \to p\,\bar{p})$ decay width, Eq. (30). We must not forget, however, that the results presented in Table I, which are at leading order in the mass corrections, are on the contrary free from the ambiguity connected with possible changes in the shape of the DA's due to higher order mass corrections, since they do not contribute to leading order.

Let us now compare our predictions for $\Gamma({}^1P_1 \to p\,\bar{p})$ with previous estimates. Assuming the resonance observed by the E760 Collaboration to be the ${}^1P_1$ state of charmonium, we have the following measurement for the product of the branching ratios $B({}^1P_1 \to p\,\bar{p})B({}^1P_1 \to J/\psi\,\pi^0)$ [1]:

$$\begin{aligned}(2.3 \pm 0.6) \times 10^{-7} \geq\ & B({}^1P_1 \to p\,\bar{p})B({}^1P_1 \to J/\psi\,\pi^0) \\ \geq\ & (1.7 \pm 0.4) \times 10^{-7} \ .\end{aligned} \tag{36}$$

The range of values in Eq. (36) corresponds to the plausible range of values for the ${}^1P_1$ total decay width, $\Gamma_T({}^1P_1)$ [1]:

$$500 \text{ keV} \leq \Gamma_T({}^1P_1) \leq 1000 \text{ keV} \ . \tag{37}$$

By using Eq. (37) and the biggest of our estimates from Table I, we find correspondingly

$$2.1 \times 10^{-5} \geq B(^1P_1 \to p\,\bar{p}) \geq 1.0 \times 10^{-5} \; . \tag{38}$$

Unfortunately we are not able to give a reliable prediction for the decay width of the process $^1P_1 \to J/\psi\,\pi^0$ in the framework of our model. Then, we cannot make a full comparison of our results with the experimental ones of Ref. [1]. A complementary situation is found in the case of the QCD multipole expansion models. In this framework, Kuang et al. [4] have predicted $\Gamma(^1P_1 \to J/\psi\,\pi^0) = 0.3\,(\alpha_{_M}/\alpha_{_E})$ keV, where $\alpha_{_E}$ and $\alpha_{_M}$ are effective electric and magnetic multipole-expansion coupling constants. There is some uncertainty on the value of the ratio $\alpha_{_M}/\alpha_{_E}$, but theoretical considerations suggest as reasonable range $1 < \alpha_{_M}/\alpha_{_E} < 3$ [4]. We thus take as indicative estimate $\alpha_{_M}/\alpha_{_E} \simeq 2$, which leads to the prediction $\Gamma(^1P_1 \to J/\psi\,\pi^0) \simeq 0.6$ keV. Comparing this result with Eqs. (36),(37), the following estimate for $B(^1P_1 \to p\,\bar{p})$ follows:

$$1.9 \times 10^{-4} \leq B(^1P_1 \to p\,\bar{p}) \leq 2.8 \times 10^{-4} \; . \tag{39}$$

The two predictions of Eqs. (38),(39) differ approximately of one order of magnitude; they are closer the smaller is the total decay width for the $^1P_1$ state. However, both theoretical calculations are at present order of magnitude estimates, and before drawing any definite conclusion better calculations are to be expected, together with improved experimental results.

Let us finally remark that our prediction for the branching ratio $B(^1P_1 \to p\,\bar{p})$, Eq. (38), compares reasonably well with the experimental branching ratios for the processes $\chi_{c2,1} \to p\,\bar{p}$, $B(\chi_{c2} \to p\,\bar{p}) = (10.0 \pm 1.0) \times 10^{-5}$, $B(\chi_{c1} \to p\,\bar{p}) = (8.6 \pm 1.2) \times 10^{-5}$ [2,22]. In fact, in QCD models the $^1P_1 \to p\,\bar{p}$ decay should be suppressed with respect to the $\chi_{c2,1} \to p\,\bar{p}$ ones by a factor $\sim m_H^2/Q^2$, being $m_H \sim 1$ GeV a typical mass scale for higher twist effects. For $Q^2 \sim M_{1,2}^2$ this should correspond to a suppression factor of about $10 - 15$, which should in turn put the expected value for $B(^1P_1 \to p\,\bar{p})$ in the range covered by our prediction.

IV. CONCLUSIONS

In this paper we have estimated the width for the decay process of the $^1P_1$ charmonium state into a proton-antiproton pair, using QCD models and including constituent quark mass corrections. In fact, massless QCD models state that this decay width is vanishing, due to the helicity selection rules. This is in possible contradiction with the recent experimental findings of the E760 Collaboration. The inclusion of higher twist effects, like constituent quark mass effects, which can still play a non negligible role at the involved energies, can in principle improve considerably the situation. However, a full, consistent treatment of these effects is out of our present possibilities. We have then used a phenomenological model which extends the massless QCD ones. In order to minimize the dependence of the quantitative results of the calculation on details of the model, we have expressed the $^1P_1 \to p\,\bar{p}$ decay width in connection with that for the $\chi_{c2} \to p\,\bar{p}$ decay process: $\Gamma(^1P_1 \to p\,\bar{p}) = [\Gamma(^1P_1 \to p\,\bar{p})/\Gamma(\chi_{c2} \to p\,\bar{p})]^{th} \times \Gamma^{exp}(\chi_{c2} \to p\,\bar{p})$. That is, we evaluate the first term in this expression in our model, at leading order in the parameter $\epsilon = m_p/M$, while

taking the available experimental measurement for the second term. This way, not only some phenomenological parameters of the model (the nucleon decay constant F_N, the value of the first derivative of the radial, $L = 1$, nonrelativistic charmonium bound-state wave function at the origin) cancel out from the result, but some other effects should be at least minimized. For example, the possible effect of Sudakov factors should be reduced. Furthermore, a possible dependence of the DA's on mass effects is also irrelevant as long as only the leading term in the $\epsilon = m_p/M$ power expansion is kept into account. A consistent inclusion of non-leading terms in the ϵ power expansion would in fact require an estimate of mass effects on the $\Gamma(\chi_{c2} \to p\,\bar{p})$ decay width and on the DA's. As shown in table I, we estimate the $\Gamma(^1P_1 \to p\,\bar{p})$ decay width to be of the order $1 - 10$ eV, depending on the DA considered. The corresponding branching ratio depends of course on the $^1P_1$ total decay width, $\Gamma_T(^1P_1)$, which is not known at present. Using, as suggested by the E760 Collaboration, the range 500 keV $\leq \Gamma_T(^1P_1) \leq$ 1000 keV, and the biggest of our estimates of Table I, we find $2.1 \times 10^{-5} \geq B(^1P_1 \to p\,\bar{p}) \geq 1.0 \times 10^{-5}$. This result is sizably lower (about one order of magnitude) than the corresponding branching ratios for the $\chi_{c1,c2}$ states, as measured by the E760 Collaboration. This is reasonable, since in QCD models the decay $^1P_1 \to p\,\bar{p}$ should be suppressed by a factor $m_H^2(\sim 1\mathrm{GeV}^2)/M_1^2 \sim 1/12$ with respect to the decays $\chi_{c1,2} \to p\,\bar{p}$. The above result is also sizably lower than that obtained by comparing the experimental results of Ref. [1] with theoretical estimates of the decay width for the process $^1P_1 \to J/\psi + \pi^0$, obtained in the framework of QCD multipole expansion models. Both our calculation and that based on QCD multiple expansion models can probably be affected by "theoretical" errors which can modify their quantitative predictions by a sizable factor. The two estimates, at least indicatively, can be taken as representative of two classes of different models: the first extending the usual perturbative QCD models by the inclusion of higher twist effects, like mass corrections; the latter including effectively some different, nonperturbative effects, like those present, e.g., in the $\eta_c \to p\bar{p}$ decay process. As such, their results should at least suggest a definite (albeit still large) range of possible values for the $\Gamma(^1P_1 \to p\,\bar{p})$ decay width, ranging approximately from some eV to some hundreds of eV.

We conclude observing once more that further, more precise, experimental measurements will certainly help in clarifying our understanding of the $^1P_1 \to p\bar{p}$ decay process and will be very useful to improve in general our theoretical models for the calculation of exclusive decay processes in the charmonium family.

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TABLES

TABLE I. Predictions, as from Eq. (34), for the decay width relative to the process ${}^1P_1 \to p\,\bar{p}$, considering several available models for the nucleon distribution amplitude (see text for more details).

DA	$\Gamma({}^1P_1 \to p\,\bar{p})$ (eV)
NR	0.0
AS	0.0
CZ	10.5
COZ	2.7
KS	1.2
GS	1.3
HET	4.7

FIGURES

FIG. 1. The Feynman diagram which, to lowest order in α_s, describes the elementary process $Q\bar{Q} \to q_1 q_2 q_3 \bar{q}_1 \bar{q}_2 \bar{q}_3$, for a quarkonium state with charge conjugation $C = -1$. In the $Q\bar{Q}$ center-of-mass frame, $c^\mu = (E, \boldsymbol{k}/2)$ and $\bar{c}^\mu = (E, -\boldsymbol{k}/2)$, where $\boldsymbol{k}$ is the relative momentum between the c and $\bar{c}$ quarks; $q_i = x_i p$ and $\bar{q}_i = y_i \bar{p}$ $(i = 1, 2, 3)$, with $p^\mu = (E, \boldsymbol{q})$, $\bar{p}^\mu = (E, -\boldsymbol{q})$ and $\boldsymbol{q} = (q \sin\theta \cos\phi, q \sin\theta \sin\phi, q \cos\theta)$. $a, b, c,$ $i, j, l, l', m_{1,2,3}$, $n_{1,2,3}$ are color indices; the λ's label helicities.

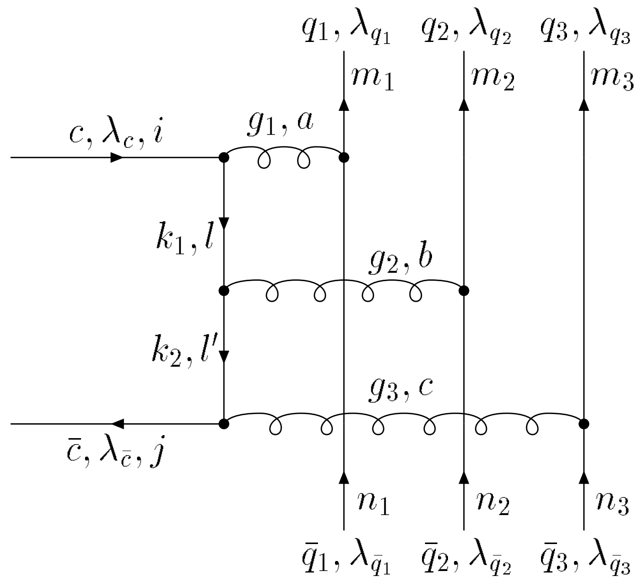
q_1, λ_{q_1}
q_2, λ_{q_2}
q_3, λ_{q_3}
m_1
m_2
m_3
c, λ_c, i
g_1, a
k_1, l
g_2, b
k_2, l'
g_3, c
$\bar{c}, \lambda_{\bar{c}}, j$
n_1
n_2
n_3
$\bar{q}_1, \lambda_{\bar{q}_1}$
$\bar{q}_2, \lambda_{\bar{q}_2}$
$\bar{q}_3, \lambda_{\bar{q}_3}$

Fig. 1